Mathematics and Madness

The pursuit of mathematics is a divine madness of the human spirit.

Alfred North Whitehead

If you watch a movie about mathematics, there's a high chance that the theme of madness will be included too. The film *Proof* featured two maths talents, a father and daughter duo played by Anthony Hopkins and Gwynneth Paltrow, both grappling with their mental health as they tried to push back the boundaries of mathematics. Other films, including *Pi, Good Will Hunting* and *A Beautiful Mind* investigate the same territory. The recently released *X + Y* touches only briefly on madness, but it does ask if doing maths to a high level pushes you into a personality where communicating with others is hard. Or is it the other way around?

There are a number of hypotheses (we might call them stereotypes) connecting maths and madness for us to consider. There's the idea that mathematicians are other-worldly, and wrapped up in their subject. Archimedes died this way, it's said; the soldier who was to murder him found the great man absorbed in diagrams in the sand. His possibly apocryphal last words were 'Do not disturb my circles!', which if true shows an admirable commitment to mathematical truth ahead of impending death. We all know what it is to get caught up with a mathematical problem that consumes us, and it's possible that mathematics does this to human beings more profoundly that with some other subjects.

There is also the vision of mathematics as an arcane revelation so powerful that it unsettles us. Andrew Wiles will not thank me for including him as an example in an article on 'madness', but his story as prover of Fermat's Last Theorem strikes me as showing how someone essentially stable can force himself into a place where his mind is under extraordinary stress. He worked on his problem alone for years; there was always the risk that he would have nothing to show for it. Cantor ran into this problem with the Continuum Hypothesis; he couldn't resolve it, and the failure was painful. What happened with Wiles? He announced his proof, but was then faced with a major flaw that was extremely hard to repair. He is candid now that doing maths in the full gaze of the mathematical community was crippling. But then - the happy ending. Wiles had his supreme revelation (you can see him on Youtube battling in silence for 13 seconds to communicate it) and his problem was solved. Cantor was not so fortunate.

Did all that solitary activity cause mental harm? Most mathematical research these days is done by people working together; the loneliness of the long-distance mathematician that Wiles put himself through is rare. Maybe a team approach introduces checks and balances that tend to preserve sanity and prevent forays into unhelpful mental country.

All discussions of mathematicians and madness run into problems of determining cause and effect. 'Madness,' whatever that is (and doctors would want to be more precise), can lead to brilliant breakthroughs (or so the argument goes); any new way of thinking mathematically requires a certain non-rational spark of originality, of thinking 'outside the box'. But then, such breakthroughs can be controversial and attract criticism, and this can in turn lead to mental distress of its own. Cantor again is an interesting example. His ideas on infinity and transfinite numbers were savagely attacked, by Poincare and Kronecker amongst others, and this hurt Cantor greatly. He clearly had an innate disposition towards depression, but would it have been triggered had he kept to more mainstream mathematics?

Cantor was a set theorist, and there is a conjecture that such mathematicians are especially vulnerable to mental instability. Perhaps sets are such nebulous objects, that are so prone to paradoxical argument, that to make set theory your life's work is asking for trouble. The book *Logicomix* investigates this theme in comic-strip form to great effect. Certainly Godel's life ended sadly, as he starved himself to death, convinced that people were trying to poison him. But then others have performed hypothesis tests to examine if significantly more set theorists have 'gone mad' than other mathematicians, and the results have been negative.

As a whimsical aside, set theory includes the idea of Maximal Almost Disjoint sets, while psychology includes Sensory, Emotional and Thought Studies. Thus set theory is a branch of psychology, and madness is part of set theory. If you are close to the edge already, this could be enough to topple you over.

But then there's an alternative vision of mathematics, one that I subscribe to, saying that rather than unsettling, mathematics can soothe the troubled mind. Through the deepest trauma, people who love maths can turn to the subject and its beautiful demands to find respite from the slings and arrows that assail them elsewhere. Mathematics as healer, a rock on which to build your life, rather than trying to construct a home on the sand of impermanence.

It's hard to examine the theme of maths and madness without mentioning God. I do understand that believing in God becomes less fashionable by the day (and given the way some of the theists amongst us behave, that's not completely surprising) but certainly if you are a person of religious faith, then your 'going mad,' if it happens, is likely to recruit symbols, ideas, themes from your faith (and your maths). Godel was a theist. Cantor saw himself as 'a messenger from God,' an alarming phrase that further complicated his condition. Ramanujan (who learnt his maths in an extremely solitary way) claimed that his results were revealed to him by his family goddess (that idea of 'revelation' again). He also memorably said, 'An equation for me has no meaning unless it expresses a thought of God'.

John Nash, the Nobel Prize winner and the subject of *A Beautiful Mind*, is thankfully still with us. He has also written memorably about the link between his work and his illness. He says, 'I wouldn't have had good scientific ideas if I had thought more normally.' He confesses that the eventual pressure to 'fit in,' in order to get his results more accepted, succeeded, but that once he had become more establishment in his thinking, he felt more limited. He had an incentive to become well, and he needed that, because as he says, 'I think madness can be an escape. If things are not so good, you maybe want to imagine something better.'

His comments on the old adage that 'Maths is a young person's game' are interesting.'I'm still making the effort at 66 and it's conceivable that, with a gap period of about 25 years of partially deluded thinking providing a sort of vacation, my situation may be atypical.'

Nash likes the idea of evolutionary psychology; that madness is a mutation, and as with all mutations, some are helpful while most are not. It could be that a helpfully mad person is the first to make the leap to a new level of consciousness for the human race.

I have a personal involvement with this topic to own up to. In my first full-time teaching post in a tough school in the East End of London I became depressed, had a manic breakdown, and was diagnosed as bipolar. It took a long time to recover, but maths was a huge help in that. My manic episodes threw up wild and wonderful mathematical ideas for me to explore, which I did with relish. No mathematical result publishable in any journal emerged, but I was left with playful links between my maths and my theology

that I still value. My last hospital visit was in 2000, so I hope I can say I've recovered now, and the following quotation expresses how I feels about my maths's role in that recovery.

The tantalizing and compelling pursuit of mathematical problems offers mental absorption, peace of mind amid endless challenges, repose in activity, battle without conflict, refuge from the goading urgency of contingent happenings, and the sort of beauty changeless mountains present to sense tried by the present-day kaleidoscope of events.

Morris Kline, Mathematics in Western Culture

And finally, I would say it's psychologically helpful to take the symbols, ideas, themes that your madness gives you, and at times of peace, build the most beautiful of these into your life. This is a picture of our patio.

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